Determination of the droplet effective size and optical depth of cloudy media from polarimetric measurements: theory

Alexander Kokhanovsky and Reiner Weichert

We present the development of a semi-analytical algorithm for optical particle sizing in disperse media. The algorithm is applied to the specific case of water clouds. However, it can be extended with minor modifications to other types of light-scattering medium. It is assumed that the optical thickness τ of the medium is large and the probability of photon absorption β is small. Thus the optical particle-sizing problem is studied in the regime of highly developed multiple light scattering. It was found that the degree of polarization in visible and near-infrared channels provides us with information both on the effective size of droplets and on the optical thickness τ. © 2002 Optical Society of America

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1. Introduction

The influence of clouds on radiative fluxes in the terrestrial atmosphere has been studied by many authors both experimentally and theoretically.[1–3] The knowledge obtained was applied to the solution of the inverse problem, namely, to the determination of the thermodynamic phase, microstructure parameters (e.g., the effective radius of droplets), single-scattering albedo, optical thickness, and liquid water path of cloudy media.[4–10]

However, cloudy media do not only absorb and scatter radiation. They can also be considered as major sources (along with aerosols, gases, and underlying surfaces) of polarized light in the terrestrial atmosphere. Clouds introduce a new property of incoming solar light, namely, the preferential direction of oscillation of the electric vector in a light beam. Generally speaking, both reflected and transmitted light becomes elliptically polarized.

The polarization arises mainly because of the single scattering of a light beam by a water droplet, an aerosol particle, or an ice crystal. Molecular scattering is another source of polarized light in the Earth's atmosphere. The phenomenon of multiple scattering, which is of particular importance for clouds, leads to the randomization of both photon directions and polarization states. This causes a decrease of light polarization because of an increase in the optical thickness of a cloud. Another effect of multiple scattering is the introduction of ellipticity in scattered light beams. It should be pointed out that the solar light scattered by a spherical water droplet becomes partially linearly polarized. This is due to the fact that the ellipticity of singly-scattered light is equal to zero for spherical scatterers if the incident light is unpolarized.[11]

Studies of polarization characteristics of solar light transmitted and reflected by cloudy media have a long history.[12–17] However, the real burst of research in this area was given by a launch of the POLDER (polarization and directionality of the Earth's reflectances) instrument on board the Japanese Advanced Earth Observing Satellite I.[18] The POLDER was able to transmit to the Earth a huge amount of information about polarization characteristics of light reflected from cloudy media, aerosols, and underlying surfaces at several wavelengths. Specifically, the first three components of the Stokes vector $\mathbf{S}_r(I, Q, U, V)$ have been measured for wavelengths λ equal to 443, 670, and 865 nm. There is no doubt that even more advanced polarimeters with wider spectral coverage will appear on board different satellites in the future, which makes further the-

A. Kokhanovsky (alexk@iup.physik.uni-bremen.de) is with the Institute of Environmental Physics, Bremen University, P.O. Box 33 04 40, D-28334 Bremen, Germany, and the Stepanov Institute of Physics, 70 Skarina Avenue, Minsk 220072, Belarus. R. Weichert is with the Institute of Particle Technology, Leibnizstrasse 19, D-38678 Clausthal-Zellerfeld, Germany.

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oretical studies of polarization characteristics of cloudy media extremely important. This is due to potential possibilities for global retrievals of the cloud microstructure,[9] the shape of particles,[18] and the optical thickness of clouds[17] on the basis of polarization measurements.

Of course, similar information can be obtained from the reflected intensity measurements. However, it could well appear that the degree of polarization can be used as a source of additional information about the cloud particle size distributions close to the top of a cloud.[9] Indeed, the high proportion of photons scattered from a thin upper layer of a cloud in the creation of light polarization is quite understandable. Multiply scattered light fluxes from deep layers are hardly polarized at all. Radiative characteristics, in contrast, represent the cloud as a whole. Thus the effective radius derived from radiative measurements is an average of large ensembles, of possibly quite different particle size distributions, presented in different parts of cloudy media.

It was emphasized by Hansen[12] 30 years ago that the fact that the polarization data refer to cloud tops may be not a drawback but an advantage. Indeed, it is easier to interpret observations that do not refer to some average values over the entire cloud depth.

The polarization characteristics of cloudy media can be studied by application of numerical codes[2,12] based on the vector radiative transfer equation solution. However, one can also use the fact that cloud fields are optically thick in most cases.[1] This allows the application of asymptotic analytical relations,[19] derived for optically thick disperse media with arbitrary phase functions and absorption. These solutions help us to clear physical mechanisms and main features behind the polarization change due to the increase of the size of droplets or the thickness of a cloud. Analytical solutions also provide an important tool for the simplification of the inverse problem.[20] They can be used, e.g., in studies of the information content of polarimetric measurements.

The drawback of the well-known vector asymptotic theory for optically thick layers[19] is related to the fact that the main equations of this theory depend on different auxiliary functions and parameters. Calculations of these functions and parameters are rather complex. This is due to their quite general applicability in terms of the single-scattering albedo.

The special case of weakly absorbing light-scattering media was considered by Kokhanovsky[21] in the framework of the modified asymptotic theory. Formulas obtained are rather simple and allow for the semi-analytical solution of the inverse problem. They can be applied to the case of water clouds. This is due to weak absorption of light by water droplets in a broad spectral range ($\sqrt{2.2}$ μm).

The main task of this paper is to apply equations, obtained in the framework of the modified asymptotic theory in Ref. 21, to the inverse problem solution, namely, to the determination of the droplet size and optical thickness of cloudy media. However, results obtained after minor changes are made can be applied to a much broader range of multiply scattering media.

2. Reflection and Transmission Matrices

Let us introduce the reflection $\hat{R}(\mu, \mu_0, \psi)$ and transmission $\hat{T}(\mu, \mu_0, \psi)$ matrices of cloudy media. Reflection and transmission matrices depend on the optical thickness τ of a cloud, microstructure of a cloud, the solar angle ϑ_0, the viewing angle ϑ, and the relative azimuth $\psi = \phi - \phi_0$, where ϕ_0 and ϕ are azimuths of the Sun and the observer, respectively. We also define $\mu_0 = \cos \vartheta_0$, $\mu = |\cos \vartheta_0|$. The importance of these matrices is due to the fact that they can be used for the calculation of the Stokes vector of reflected $\mathbf{S}_r(I, Q, U, V)$ and transmitted $\mathbf{S}_t(I', Q', U', V')$ light under arbitrary illumination of a cloudy layer. Namely, it follows for plane-parallel cloudy layers[15]:

$$\mathbf{S}_r(\mu, \phi) = \frac{1}{\pi}\int_0^1 \mathrm{d}\mu'\mu' \int_0^{2\pi} \mathrm{d}\phi' \hat{R}(\mu, \mu', \phi - \phi')\mathbf{S}_0(\mu', \phi'), \tag{1}$$

$$\mathbf{S}_t(\mu, \phi) = \frac{1}{\pi}\int_0^1 \mathrm{d}\mu'\mu' \int_0^{2\pi} \mathrm{d}\phi' \hat{T}(\mu, \mu', \phi - \phi') \times \mathbf{S}_0(\mu', \phi') + \mathbf{S}_0(\mu, \phi)\exp\left(-\frac{\tau}{\mu_0}\right), \tag{2}$$

where $\mathbf{S}_0(\mu', \phi')$ represents the Stokes vector of the incident light beam. The second term in Eq. (2) is small for most cloudy media ($\tau \gg 1$) and can be neglected.

In a good approximation, the Sun is a source of light incident from only one direction μ_0, ϕ_0. Then it follows:

$$\mathbf{S}_0(\mu, \phi) = \frac{1}{\mu_0}\delta(\mu - \mu_0)\delta(\phi - \phi_0)\pi\mathbf{F}, \tag{3}$$

where a vector $\mathbf{F}$ is normalized so that the first element of $\pi\mathbf{F}$ (namely, πF_1) is the net flux of solar beam per unit area of a cloud layer. Equations (1) and (2) simplify if one accounts for Eq. (3):

$$\mathbf{S}_r(\mu, \phi) = \hat{R}(\mu, \mu_0, \phi - \phi_0)\mathbf{F}, \tag{4}$$

$$\mathbf{S}_t(\mu, \phi) = \hat{T}(\mu, \mu_0, \phi - \phi_0)\mathbf{F}, \tag{5}$$

where only the first element of the column vector

$$\mathbf{F} = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix} \tag{6}$$

differs from zero for unpolarized solar radiation. Thus only elements $R_{j1}, T_{j1} (j = 1, 2, 3, 4)$ are relevant to studies of solar light propagation, scattering, and polarization in terrestrial atmosphere. Other elements are of interest only in the case of polarized (e.g., laser) incident beams.

One can see that the knowledge of matrices $\hat{R}(\mu, \mu_0, \phi - \phi_0)$ and $\hat{T}(\mu, \mu_0, \phi - \phi_0)$ is of primary importance for understanding radiative and polarization characteristics of light fluxes that emerge from a cloud body. As a matter of fact, they are Mueller matrices[22,23] for turbid media. Note that Eq. (4) can be written in the scalar form

$$I = R_{11}F_1, \quad Q = R_{21}F_1, \quad U = R_{31}F_1, \quad V = R_{41}F_1 \tag{7}$$

for the unpolarized incident light. Thus the intensity of the reflected light is determined by the element R_{11} of the reflection matrix. This is the only element that has been a major concern of most cloud optics studies up to this date. The degree of polarization is given by

$$P = \sqrt{P_l^2 + P_c^2}, \tag{8}$$

where

$$P_l = \sqrt{R_{21}^2 + R_{31}^2}/R_{11} \tag{9}$$

is the degree of linear polarization and

$$P_c = R_{41}/R_{11} \tag{10}$$

is the degree of circular polarization. One can show[24] that for particular viewing geometries (e.g., $\mu_0 = 1, \mu = 1, \phi - \phi_0 = 0, \pi$) the values of R_{31} and R_{41} are equal to zero and light is partially linearly polarized. The degree of circular polarization P_c is equal to zero in this case. It is also common to use a different definition of the degree of polarization, namely,

$$P = -(Q/I) \tag{11}$$

for linearly polarized light beams. It follows from Eq. (11) at the nadir-viewing geometry ($R_{31} = R_{41} = 0$) that

$$P = -(R_{21}/R_{11}). \tag{12}$$

We will use this definition of the degree of polarization in this study. The second component of the Stokes vector Q is proportional to the difference $I_l - I_r$, where values of I_l and I_r mean the intensities of light polarized parallel and perpendicular to a meridional plane that contains the normal to a layer and viewing direction. Thus light polarized perpendicular to the meridional plane is characterized by positive polarization. Note that the sign of the degree of polarization strongly depends on the observation geometry. For instance, it is positive in the main rainbow region (see Fig. 1). It could be both positive and negative in the glory-scattering region (see Fig. 1). Results presented in Fig. 1 were obtained by use of the Mie theory for polydispersions of water droplets at the wavelength λ equal to 443 nm. The particle size distribution was given by the following equation:

$$f(a) = Aa^{\nu} \exp[-(\nu + 3)a/a_{\mathrm{ef}}], \tag{13}$$

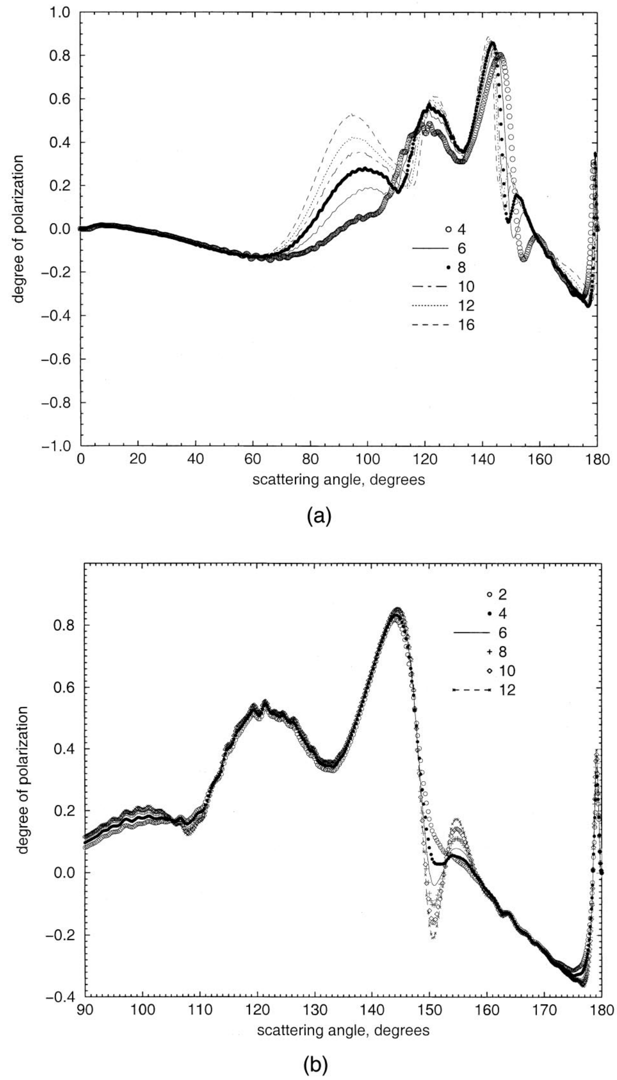


Fig. 1. Degree of polarization of light singly scattered by a polydispersion of spherical water droplets characterized by the gamma particle size distribution [Eq. (13)] with the half-width parameter $\nu = 6$ and the effective radius $a_{\mathrm{ef}} = 4$, 6, 8, 10, and 16 μm as the function of the scattering angle at $\lambda = 443$ nm. (b) Same as in (a) but at $a_{\mathrm{ef}} = 6$ μm and values of the half-width parameter $\nu = 2$, 4, 6, 8, 10, and 12.

where a_{ef} is the effective radius of droplets, ν is the half-width parameter, and

$$A = \frac{\nu^{\nu+1}}{a_0^{\nu+1}\Gamma(1 + \nu)}.$$

Note that the mode radius a_0 is related to the effective radius with the following equation:

$$a_0 = a_{\mathrm{ef}} \Big/ \left(1 + \frac{3}{\nu}\right).$$

It should be pointed out that the Stokes vector elements I, Q, U, V [see Eq. (7)] can be used for the determination of polarization ellipse parameters such as the ellipticity and the angle of preferential

oscillations of the electric vector in a light beam under study.[22]

Thus reflection and transmission matrices allow the determination of both the intensity and the polarization states of reflected and transmitted light beams under arbitrary illumination of a cloud layer.

3. Modified Asymptotic Theory

A. General Equations

The optical thickness of water clouds is large in most cases.[1] Some mixed and crystalline clouds are also extended in the vertical direction for many hundreds of meters and could be optically thick.[25] Thus the asymptotic vector theory[19,26] is quite applicable to cloudy media. The reflection matrix $\hat{R}(\mu, \mu_0, \psi, \tau)$ and transmission matrix $\hat{T}(\mu, \mu_0, \tau)$ of a homogeneous plane-parallel weakly absorbing turbid layer of a large optical thickness $\tau = \sigma_{\text{ext}}L$($\sigma_{\text{ext}}$ is the extinction coefficient and L is the geometrical thickness of a layer) can be written in the following form[21]:

$$\hat{R}(\mu, \mu_0, \psi, \tau) = \hat{R}_\infty(\mu, \mu_0, \psi) - \hat{T}(\mu, \mu_0, \tau) \times \exp(-x - y), \tag{14}$$

$$\hat{T}(\mu, \mu_0, \tau) = t\mathbf{K}_0(\mu)\mathbf{K}_0^T(\mu_0), \tag{15}$$

where

$$\begin{aligned} &x = k\tau, \qquad y = 4qk, \qquad q = [3(1 - g)]^{-1}, \\ &k = [3(1 - g)(1 - \omega_0)]^{1/2}, \\ &\omega_0 = 1 - \beta, \qquad \beta = \frac{\sigma_{\text{abs}}}{\sigma_{\text{ext}}}, \\ &\tau = \sigma_{\text{ext}}L, \end{aligned} \tag{16}$$

$\hat{R}_\infty(\mu, \mu_0, \psi)$ is the reflection matrix for a semi-infinite medium with the same local optical properties as a finite one, L is the geometrical thickness of the medium, σ_{ext} is the extinction coefficient, σ_{abs} is the absorption coefficient, $g = \frac{1}{2}\int_0^\pi p(\theta)\sin\theta\cos\theta d\theta$ is the asymmetry parameter of the phase function $p(\theta)$, and θ is the scattering angle. Note that the vector $\mathbf{K}_0^T(\mu_0)$ in Eq. (15) is transposed to the vector $\mathbf{K}_0(\mu_0)$. The vector $\mathbf{K}_0(\mu)$ also occurs in the so-called Milne problem.[3] It describes the angular dependence and the polarization characteristics of light emerging from semi-infinite turbid layers with sources of radiation located deep inside the medium. The value of

$$t = \frac{\sinh y}{\sinh(x + \alpha y)} \tag{17}$$

in Eq. (15) is the global transmittance,[21] defined as[3]

$$t = 4\int_0^1 \mu d\mu \int_0^1 \mu_0 d\mu_0 T_{11}(\mu, \mu_0, \tau). \tag{18}$$

Equation (18) also follows from Eq. (15), taking into account the integral relation[3]

$$2\int_0^1 \mu d\mu K_{01}(\mu) = 1, \tag{19}$$

where $K_{01}(\mu)$ is the first component of the escape vector $\mathbf{K}_0(\mu)$. The value of α is approximately equal to 1.07.[24]

Equations (14) and (15), derived in Ref. 21 for small probabilities of photon absorption, are much simpler than initial asymptotic formulas, obtained by Domke.[19] They allow us to calculate the reflection and transmission matrices of thick weakly absorbing disperse media by a simple means if the solution of the problem for a semi-infinite medium with the same phase matrix as for a finite layer under consideration is available.[24,26] This reduction of a general problem to the case of a semi-infinite medium is of general importance for the radiative transfer theory.

B. Nonabsorbing Media

It follows from Eqs. (12), (14), and (15) for the degree of polarization of the reflected light at the nadir observation in the case of nonabsorbing media[21]:

$$P(\mu_0) = \frac{P_\infty{}^0(\mu_0)}{1 - tU(\mu_0)}, \tag{20}$$

where

$$U(\mu_0) = \frac{K_{01}(1)K_{01}(\mu_0)}{R_{\infty 11}(\mu_0, 1)}, \tag{21}$$

and we have instead of Eq. (17) at $\beta = 0$:

$$t = \frac{1}{\alpha + 0.75\tau(1 - g)}. \tag{22}$$

The value of

$$P_\infty{}^0(\mu_0) = -\frac{R_{\infty 21}{}^0(\mu_0, 1)}{R_{\infty 11}{}^0(\mu_0, 1)} \tag{23}$$

is the degree of polarization for a semi-infinite nonabsorbing medium at the nadir observation. Here $R_{\infty 11}{}^0(\mu_0, 1)$ and $R_{\infty 21}{}^0(\mu_0, 1)$ are correspondent elements of the reflection matrix [see Eq. (4)] of a nonabsorbing semi-infinite medium. Note that we derived Eq. (20) by taking into account the equality[26] $K_{02}(1) = 0$. Owing to this equality, the second component of the Stokes vector for optically thick media coincides with the value of Q for semi-infinite layers at the nadir observation geometry. This fact can be used for the retrieval of the size of particles from spectral measurements of $Q(\lambda)$ even if the precise information on the optical thickness is not available. Thus an accurate estimation of the effective radius of droplets can even be obtained from measurements of Q at a single wavelength in the visible.

Both functions $K_{01}(\mu_0)$ (see Fig. 2) and $R_{\infty 11}{}^0(\mu_0, 1)$ (see Zege *et al.*[27], Fig. 3.3b) only slightly depend on

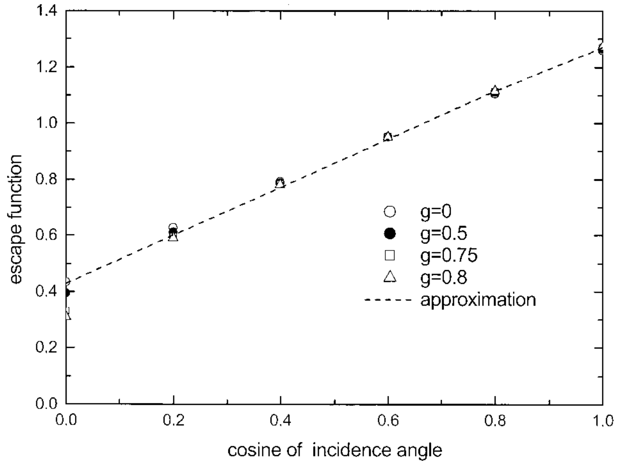


Fig. 2. Dependence of the escape function K_{01} on the cosine of the incidence angle for the Henyey–Greenstein phase function with the asymmetry parameter $g = 0, 0.5, 0.75, 0.8$.[28] Results of calculations with Eqs. (24) are presented by a dashed line.

the size of scatterers. They can be represented, e.g., by the following approximate formulas[27]:

$$K_{01}(\mu_0) = \frac{3}{7}(1 + 2\mu_0), \qquad R_{\infty 11}{}^0(\mu_0, 1) = \frac{1 + 4\mu_0}{2(1 + \mu_0)}. \tag{24}$$

Thus it follows from Eqs. (21) and (24):

$$U(\mu_0) = \frac{54(1 + 2\mu_0)(1 + \mu_0)}{49(1 + 4\mu_0)}. \tag{25}$$

The dependence $U(\vartheta_0)$ is presented in Fig. 3. One can see that $U \in [1.03, 1.33]$. Thus the function $U(\vartheta_0)$ does not change drastically with the angle and can be substituted by a constant value (e.g., $U = 1.18$). Note that the function $U(\mu_0)$ is multiplied by the global transmittance t in Eq. (20), the latter being a small number for the thick clouds considered here. Thus one can see that small errors in the function $U(\mu_0)$ do not much influence the degree of polarization calculated with Eq. (20).

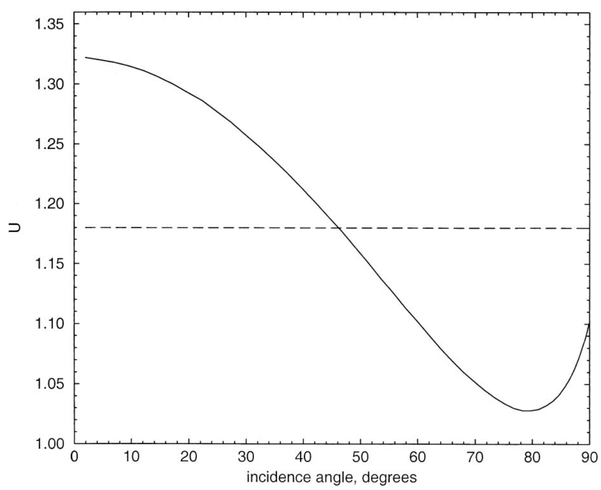


Fig. 3. Dependence $U(\vartheta_0)$ according to Eq. (25). The dashed line represents the average value of $U = 1.18$.

Thus approximate equations (20), (22), (25), and (17) allow us to reduce the complexity of calculations of the function $P(\mu_0)$. Note that exact calculations require the solution of the four coupled integro-differential equations for the Stokes vector components of the vector radiative transfer theory,[3] which is a far more complex task than application of Eq. (20).

In more general cases [see Eqs. (14) and (15)] the functions $\mathbf{K}_0(\mu)$ and $\hat{R}_\infty{}^0(\mu, \mu_0, \psi)$ for a nonabsorbing semi-infinite light-scattering medium should be found numerically. They depend on the phase matrix of a random medium in question. However, they do not depend on the single-scattering albedo ω_0 and the optical thickness τ by definition. Another interesting feature of these functions is their weak dependence on the microstructure parameters (size, shape, and chemical composition of particles) of a medium under study in the broad range of angular parameters μ, μ_0, ψ. This is due to the randomization of both polarization states and propagation directions of photons in highly scattering nonabsorbing semi-infinite layers irrespective of the local optical properties of the medium in which they propagate.

C. Weakly Absorbing Media

Equation (20) is easily generalized to account for the absorption of light in a cloudy medium. Namely, it follows from Eqs. (12) and (14)[21]:

$$P(\mu_0) = \frac{P_\infty{}^*(\mu_0)}{1 - U^*(\mu_0)t\,\exp(-x - y)}, \tag{26}$$

where the global transmittance t is given by Eq. (17) and

$$U^*(\mu_0) = \frac{K_{01}(1)K_{01}(\mu_0)}{R_{\infty 11}{}^*(\mu_0)}. \tag{27}$$

Values of $P_\infty{}^*(\mu_0)$ and $R_{\infty 11}{}^*(\mu_0, 1)$ represent the degree of polarization and the reflection function of a semi-infinite weakly absorbing cloud at the nadir-viewing geometry. They can be expressed via the correspondent functions for nonabsorbing media. Indeed, it follows for the reflection function $R_\infty \equiv R_{\infty 11}{}^*$[27]:

$$R_\infty(\mu, \mu_0, \psi) = R_\infty{}^0(\mu, \mu_0, \psi)\exp[-yU(\mu, \mu_0, \psi)], \tag{28}$$

where

$$U(\mu, \mu_0, \phi) = K_0(\mu)K_0(\mu_0)/R_\infty{}^{0-1}(\mu_0, \mu, \phi), \tag{29}$$

$K_0 \equiv K_{01}$, and $R_\infty{}^0$ is the reflection function of the semi-infinite nonabsorbing light-scattering medium.

Our calculations show that the accuracy of Eq. (28) can be improved at $y > 1$ if we substitute the value of

U by $s = (1 - 0.05y)U$. Thus we have for the value of $R_{\infty 11}{}^*(\mu_0, 1)$ in Eq. (27):

$$R_{\infty 11}{}^*(\mu, \mu_0, \psi) = R_\infty{}^0(\mu, \mu_0, \psi)\exp(-sy). \quad (30)$$

Substituting Eqs. (27) and (30) into Eq. (26), we obtain

$$P(\mu_0) = \frac{P_\infty{}^*(\mu_0)}{1 - U(\mu_0)t\, \exp[-x - y(1 - s)]}, \quad (31)$$

where we also used Eq. (21).

Let us find now the relationship between functions $P_\infty{}^*(\mu_0)$ and $P_\infty{}^0(\mu_0)$. For this we will use Eq. (12) and the fact that the value of $R_{\infty 21}{}^*(\mu_0)$ for semi-infinite weakly absorbing media does not differ much from the same value for the nonabsorbing case.[24] Then we have by using Eqs. (12) and (30)

$$P_\infty{}^*(\mu_0) = P_\infty{}^0(\mu, \mu_0, \psi)\exp(sy). \quad (32)$$

4. Inverse Problem

A. Algorithm Description

Equations (20) and (31) can be used for the semi-analytical retrieval of droplet sizes from the two-wavelength polarimetric inversion algorithm. Let us rewrite these equations here, explicitly showing the wavelength dependence:

$$P(\lambda_1) = \frac{P_\infty{}^0(\lambda_1)}{1 - Ut(\lambda_1)}, \quad (33)$$

$$P(\lambda_2) = \frac{P_\infty{}^0(\lambda_2)\exp[s(\lambda_2)y(\lambda_2)]}{1 - Ut(\lambda_2)\exp\{-x(\lambda_2) - y(\lambda_2)[1 - s(\lambda_2)]\}}, \quad (34)$$

where λ_1 is the wavelength in the visible and λ_2 is the wavelength in the infrared. We also accounted for Eq. (32). A closer look at these equations shows that values $P(\lambda_1)$ and $P(\lambda_2)$ depend only on the solar elevation, the optical thickness at each wavelength, the droplet size distribution, and the refractive index. It is known, however, that the influence of the type of the particle size distribution on the integral light-scattering characteristics, such as x and y in Eqs. (33) and (34), is weak.[29] So we will neglect this dependence and characterize the size of droplets by just one number—the effective radius of droplets. We also ignore the possible variability of the half-width of the droplet size distribution and use only the distribution [Eq. (13)] at $\nu = 6$ in the discussion, which follows. The dependence of the degree of polarization of the singly scattered light on the value of ν is presented in Fig. 1(b). We see that this dependence can be neglected for most scattering angles. It is significant, however, at $\theta \approx 150$–155 degrees.

The parametrizations for the integral light-scattering characteristics [Eqs. (16)] as well as functions $P_\infty{}^0(\lambda_1)$, $P_\infty{}^0(\lambda_2)$ at wavelengths $\lambda_1 = 0.65$ μm and $\lambda_2 = 1.55$ μm are presented in Appendices A and B. We will use these two wavelengths in the development of the inversion algorithm.

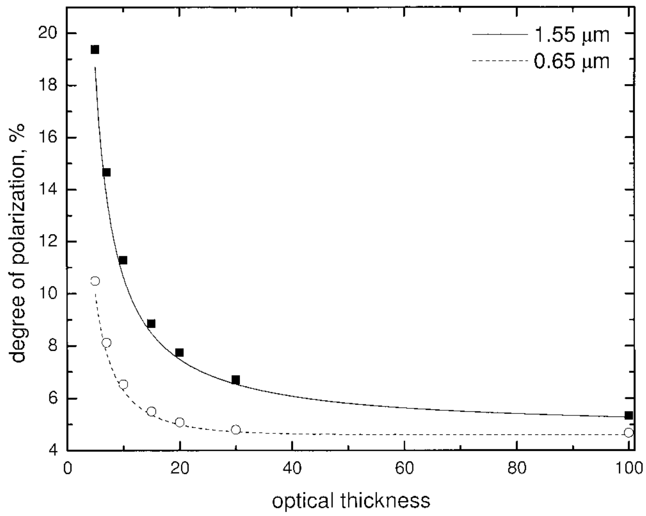


Fig. 4. Dependence of the degree of polarization on the optical thickness at $\lambda = 0.65$ μm, 1.55 μm, $\vartheta_0 = 37°$, $\vartheta = 0°$, $a_{ef} = 6$ μm, and $\nu = 6$ [see Eq. (13)] according to the exact radiative transfer calculations (symbols) and Eqs. (33), (34), and (B1).

Thus we have the system of the two Eqs. (33) and (34) to find the optical thickness in the visible and the size of droplets. We proceed as follows. First of all, we express the value $\tau(\lambda_1)$ in Eq. (33) via the measured degree of polarization $P(\lambda_1)$:

$$\tau(\lambda_1) = \frac{4[t^{-1}(\lambda_1) - \alpha]}{3[1 - g(\lambda_1)]}, \quad (35)$$

where

$$t(\lambda_1) = \frac{1}{U}\left[1 - \frac{P_\infty{}^0(\lambda_1)}{P(\lambda_1)}\right], \quad (36)$$

and we also used Eq. (22). Second, we use the fact that the ratio $\Upsilon(a_{ef}) = \tau(\lambda_2)/\tau(\lambda_1)$ depends only on the effective radius of droplets (see Appendix A). So we can substitute the value of $\tau(\lambda_2)$ by $\Upsilon(a_{ef})\tau(\lambda_1)$ in Eq. (34). It gives us a single transcendent equation [Eq. (34)] for the effective radius determination. This equation can be easily solved with standard techniques. The value of $\tau(\lambda_1)$ can be obtained from Eq. (35) afterward. This completes the proposed algorithm description.

B. Accuracy of the Retrieval Algorithm

Let us study the errors associated with the retrieval algorithm proposed. We will use the fixed viewing geometry ($\vartheta_0 = 37°$, $\vartheta = 0°$), in which the sensitivity of the measured degree of polarization in the visible to the value of the effective radius of droplets is high (see Fig. 1). However, one can also choose another geometry. This is mostly due to the fact that the information on sizes of droplets is contained in the parameter y, which does not depend on the observation geometry. This is an important point.

First, let us study the accuracy of Eqs. (33) and (34). The results of comparisons with the exact ra-

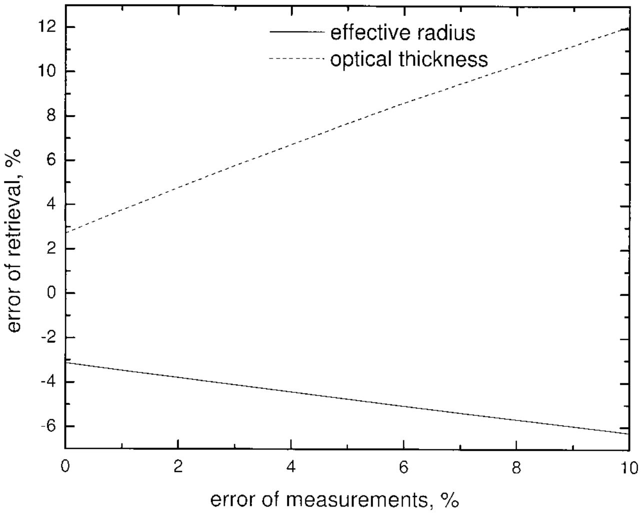


Fig. 5. Error of the retrieval as the function of the errors of measurements.

diative transfer code are presented in Fig. 4. One can see that errors of approximate formulas at optical thicknesses larger than 5 are smaller than possible measurement and forward-model assumption errors. So approximate equations can be applied to the inverse problem solution.

Let us consider now the error in the retrieval of a_{ef} and τ that is due to possible measurement errors. For this, we have calculated the values of $P(\lambda_1)$ and $P(\lambda_2)$ at $a_{ef} = 6$ μm and $\tau = 10$ with the exact radiative transfer code for wavelengths 0.65 and 1.55 μm. Then we assumed that the actual values of $P(\lambda_1)$ and $P(\lambda_2)$ differ from the calculated values by the relative positive error Δ, which was changed from 0% to 10%. The results are presented in Fig. 5. We see that the errors in the retrieved values of a_{ef} and τ are smaller than 12% in this case. This shows that our algorithm is not particularly sensitive to the measurement errors at $a_{ef} = 6$ μm and $\tau = 10$. Clearly, the errors of retrieval of the optical thickness increase with the optical thickness. This is due to the fact that the degree of polarization for extremely thick clouds is not dependent on the optical thickness at all [see Eqs. (33) and (34) at $t = 0$]. So it cannot be retrieved in principle. This is not the case as far as the effective radius of droplets is concerned. It can be derived for clouds of arbitrarily large thicknesses. However, for large clouds the transmission t is approximately equal to zero and the transcendent equation [Eq. (34)] transforms into more simple equation [Eq. (32)].

Solving this single transcendent equation numerically, one obtains the value of a_{ef}, which subsequently can be used to find the optical thickness and liquid water path of clouds, if they are not thick. Clouds with optical thicknesses around 15 and larger could be considered as semi-infinite for wavelengths larger than 1.55 μm. Of course, this is not the case in the visible. Knowing the values of a_{ef} and τ, one can find it easy to obtain many other parameters of cloudy media, including their transmittance, reflection function, and spherical albedo.

Spectral measurements at several wavelengths can improve the accuracy of the algorithm. However, remarkably, as we have discussed before, there is even a possibility[17] of finding the size of particles and the liquid water path of clouds from measurements of the reflection function and the degree of polarization at the single wavelength in the visible. For this, one should make measurements at the rainbow geometry.

5. Conclusions

We presented here a system of simple equations that can be used for parametrizations of radiative and polarization characteristics of cloudy media. The algorithm of the inverse problem solution was proposed. It is based on spectral measurements of the degree of polarization of reflected light in the visible and the infrared at the rainbow geometry. Note that the method proposed is not significantly influenced by the choice of the observation geometry. This is not the case, e.g., when only measurements in the visible are used.[9,17]

The reflection of light from the surface and scattering by molecules and aerosols was neglected in this study. However, they can easily be taken into account. The method described can also be used for multiple light-scattering disperse media other than water clouds. It is also applicable with slight modifications for the retrieval of the spectral dependence of the imaginary part of the refractive index of droplets, which can differ from the refractive index of pure water because of various sources of contamination.

Appendix A: Local Optical Characteristics of Cloudy Media

We use the parametrizations of the extinction coefficient σ_{ext}, the asymmetry parameter g, and the absorption coefficient σ_{abs} of cloudy media:

$$\sigma_{ext} = \frac{3C}{2a_{ef}}\left[1 + \frac{1.1}{(x_{ef})^{2/3}}\right], \tag{A1}$$

$$\sigma_{abs} = \frac{4\pi\chi C}{\lambda}\sum_{n=0}^{4} p_n(x_{ef})^n, \tag{A2}$$

$$1 - g = \sum_{n=0}^{4} q_n(x_{ef})^{-2n/3}. \tag{A3}$$

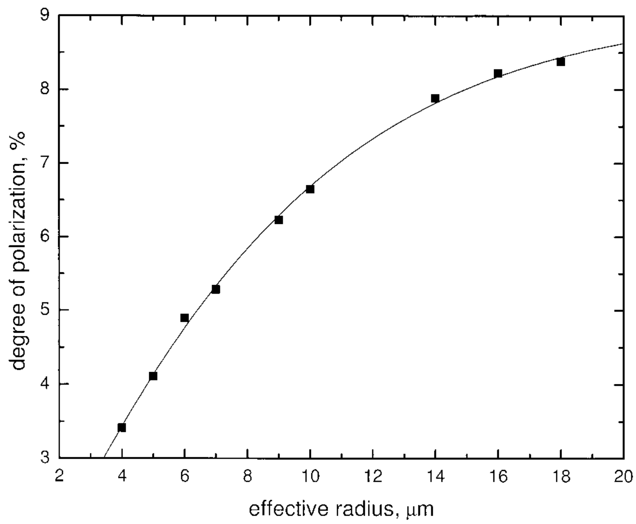
Here C is the volumetric concentration of droplets, $x_{ef} = 2\pi a_{ef}/\lambda$, χ is the imaginary part of the refractive index of particles. The parameters p_n at the 1.55-μm wavelength, where there is an appreciable amount of light absorption by water droplets, are $p_0 = 1.671$, $p_1 = 0.0025$, $p_2 = -2.365 \times 10^{-4}$, $p_3 = 2.861 \times 10^{-6}$, and $p_4 = -1.05 \times 10^{-8}$. The parameters q_n depend on the refractive index of particles. They are given in Table 1 for water droplets at the 0.65- and 1.55-μm wavelengths. The accuracy of Eqs. (A1)–(A3) is better than 3% for the effective

Table 1. Parameters q_n for Different Wavelengths λ

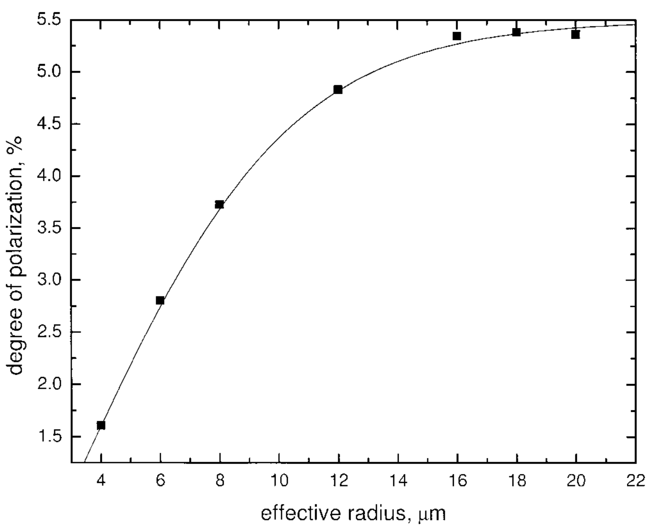
λ, μm	q_0	q_1	q_2	q_3	q_4
0.65	0.1121	0.5118	0.8997	0.0	0.0
1.55	0.0608	2.465	−32.98	248.94	−636

Table 2. Parameters in Eq. (A4) at the Wavelengths 0.65 and 1.55 μm

λ, μm	α	ς	ζ	σ
0.65	9.1	22.0	0.37	0.17
1.55	5.5	8.4	−1.0	0.29



(a)



(b)

Fig. 6. Degree of polarization, calculated with Eq. (B1) (line) and exact radiative transfer code (symbols) at (a) $\lambda = 0.65$ μm, (b) 1.55 μm. Other parameters are $\beta = 0$, $\vartheta_0 = 37°$, $\vartheta = 0°$, $a_{\mathrm{ef}} = 6$ μm, and $\nu = 6$.

radius of droplets in the range of 4–16 μm, which is most common for water clouds.

Clearly, the ratio of optical thicknesses Y for selected wavelengths is given by ratios of values $\{1 + [1.1/(x_{\mathrm{ef}})^{2/3}]\}$ for both wavelengths.

Appendix B: Degree of Polarization at the Rainbow Angle

We present here the results of the parametrization of the degree of polarization at the rainbow geometry. It is assumed that the incidence angle is equal to 37 deg and the observation angle is zero, which means the nadir observation. Also, we consider the case of a semi-infinite nonabsorbing medium. Then the degree of polarization depends only on the effective radius of droplets at this geometry (see Fig. 1). It can be presented by the following equation:

$$P_\infty{}^0 = \alpha - \frac{\varsigma}{1 + \exp(\zeta + \sigma a_{\mathrm{ef}})}, \qquad \text{(B1)}$$

where the parameters α, ς, ζ, σ are presented in Table 2 at the 0.65- and 1.55-μm wavelengths. The accuracy of these equations is better than 6% at the effective radii in the range of 4–16 μm (see Fig. 6).

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